Using the Fourth Eigenvalue of the Covariance Matrix for Assessing the Quality of Polarimetric SAR Data

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***Abstract*—In this paper we show that the eigen-decomposition of the covariance matrix can be used to assess the quality of polarimetric SAR data. The fourth eigenvalue, l_4, is a measure for the noise in the image, independent of image calibration. Misregistration of the two cross-polarization channels leads to image content appearing in the l_4 image and is easily detectable. No sensitivity to cross-talk and channel imbalance is present. We show quality dependencies using simulated data and present an example of airborne data where we successfully used the l_4 test to identify a quality issue.**

Keywords- SAR polarimetry, eigenvalue analysis, data quality

I. INTRODUCTION

Quantitative analysis of SAR data requires the input data to be calibrated. For SAR polarimetry, data calibration is particularly important. In addition to per-channel calibration, inter-channel relationships like channel phase, amplitude ratios and channel correlation need to be taken into consideration.

While the main responsibility of quality assurance rests with the data provider, users should test data for quality. If calibration targets like corner reflectors or transponders are available in the scene, a quantitative test is possible using a calibration method [1]. Without such targets, a qualitative check of polarimetric parameters is a possible option.

The objective of this paper is to evaluate the potential of the fourth eigenvalue of the covariance matrix to assess the quality of polarimetric SAR data. This is one step toward the development of a comprehensive test for data quality assessment.

The remainder of this paper is organized as follows. Section II describes the eigenanalysis of the coherency and covariance matrices. In Section III, a data simulation and de-calibration scheme is described. In Section IV, simulation results are presented and dependencies of λ_4 are evaluated. Examples from airborne polarimetric data are provided in Section V.

II. EIGENANALYSIS OF COHERENCY AND COVARIANCE MATRIX

One frequently used analysis method for distributed polarimetric data is the Cloude/Pottier decomposition [2]. This method is based on the eigen-analysis of the 3×3 coherency matrix, **T**. **T** can be expressed in the form of eigenvectors, $\mathbf{e_i}$, and eigenvalues, λ_i, which for the 4×4 case is

$$\mathbf{T} = \mathbf{U_3} \begin{bmatrix} l_1 & 0 & 0 & 0 \\ 0 & l_2 & 0 & 0 \\ 0 & 0 & l_3 & 0 \\ 0 & 0 & 0 & l_4 \end{bmatrix} \mathbf{U_3}^{+}$$

$$\text{where} \quad \mathbf{U_3} = \left[\underline{\mathbf{e}_1} \quad \underline{\mathbf{e}_2} \quad \underline{\mathbf{e}_3} \quad \underline{\mathbf{e}_4} \right]$$

$$\text{and} \qquad l_1 \geq l_2 \geq l_3 \geq l_4 .$$

Assuming reciprocity (VH=HV), the 4×4 matrix is rank 3. The fourth eigenvalue, λ_4, can then be expected to be a representation of the noise in the image [3].

The covariance matrix, **C**, shares the same eigenvalues but has different eigenvectors. For the quality assessment described here, either T or C can be used.

III. DATA SIMULATION AND DE-CALIBRATION

A quantitative assessment of the sensitivity of λ_4 to data quality is performed using data simulation to create a controlled environment. The simulation of calibrated polarimetric data is based on a total of 6 parameters:

- 3 backscatter strengths (HH, VV, HV)
- the correlation magnitude ($|\rho|$) and the phase (ϕ) between HH and VV
- the incidence angle (δ_i)

The parameters are estimated from a real scene, which had previously been the subject of classification experiments using k classes.

Let z_1, z_2, and z_3 be complex Gaussian variables with zero mean and a variance of one. Based on this definition,

$$z_1 \cdot z_1^* = 1\,;\; z_2 \cdot z_2^* = 1\,;\; z_1 \cdot z_2^* = 0\,;\; z_2 \cdot z_1^* = 0.$$

If we define

$$u_1 = \frac{1}{\sqrt{2}} \cdot \left(\sqrt{1+|r|} \cdot z_1 + \sqrt{1-|r|} \cdot z_2 \right)$$
$$u_2 = \frac{1}{\sqrt{2}} \cdot \left(\sqrt{1+|r|} \cdot z_1 - \sqrt{1-|r|} \cdot z_2 \right)$$

the covariance matrix $\mathbf{R_u}$ of the vector $\mathbf{u}$ is

$$R_u = u \cdot u^+ = \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}.$$

The covariance matrix for symmetrized polarimetric data is

$$C = \begin{pmatrix} |HH|^2 & 0 & \sqrt{|HH|^2 \cdot |VV|^2} \cdot |r| e^{-jf} \\ 0 & 2|HV|^2 & 0 \\ \sqrt{|HH|^2 \cdot |VV|^2} \cdot |r| e^{jf} & 0 & |VV|^2 \end{pmatrix}$$

Using this analogy, we can now define

$$HH = |HH|_{est} \cdot \frac{1}{\sqrt{2}} \left(\sqrt{1+|r|} \cdot z_1 + \sqrt{1-|r|} \cdot z_2 \right)$$
$$VV = |VV|_{est} \cdot e^{jf} \cdot \frac{1}{\sqrt{2}} \left(\sqrt{1+|r|} \cdot z_1 - \sqrt{1-|r|} \cdot z_2 \right)$$
$$HV = \sqrt{2} \cdot |HV|_{est} \cdot z_3$$

The above values are single look estimates that do not contain any noise. To add noise, complex Gaussian variables n_1, n_2, n_3, and n_4 are weighted by the system noise level and added to the signal

$$HH_N = HH + NESZ * n_1$$
$$HV_N = HV + NESZ * n_2$$
$$VH_N = HV + NESZ * n_3$$
$$VV_N = VV + NESZ * n_4.$$

The data simulation also takes incidence angle dependent variation of the parameters into account. For this purpose the swath is divided into 20 range sections and the required parameters are estimated in each section. Based upon these 20 estimates per parameter, a second-order polynomial is used to approximate the incidence angle dependency of the parameter over the swath. This results in simulation parameters changing for each range line.

The simulation process can then be summarized as follows:

<u>(i) Parameter estimation from real data</u>

1. Calculate the polarimetric parameters for a scene.
2. Classify the scene into k classes (k is user defined)
3. FOR each class
   - Estimate the incidence angle dependency of each parameter by dividing the image into 20 range sections and use a second order polynomial to calculate an estimate for each range cell.

   END

<u>(ii) Simulation</u>

4. FOR each class
   - FOR each range line
     - Simulate data based on the estimated parameters as described in this section

     END

   END
5. Add noise
6. Write the file to disk

To generate data with controlled calibration errors, we used the following model [4]:

$$\mathbf{Z} = \mathbf{R.S.T}_r + \mathbf{N}$$

where $\mathbf{Z}$ is the measured scattering matrix and $\mathbf{S}$ is the actual scattering matrix.

$$\mathbf{Z} = \begin{pmatrix} Z_{HH} & Z_{HV} \\ Z_{VH} & Z_{VV} \end{pmatrix} \;;\; \mathbf{S} = \begin{pmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{pmatrix}$$

$\mathbf{R}$ and $\mathbf{T_r}$ are the distortion matrices for the transmit and receive channels, respectively.

$$\mathbf{R} = g_r \begin{pmatrix} 1 & d_4 \\ d_3 & f_r \end{pmatrix} \;;\; \mathbf{T}_r = g_t \begin{pmatrix} 1 & d_1 \\ d_2 & f_t \end{pmatrix}$$

The parameters, g_r and g_t are gains that were set to 1 in this experiment. The complex-valued f terms represent the H/V imbalances and the δ values represent the complex-valued cross talk terms.

$\mathbf{N}$ is the noise matrix and is written as

$$\mathbf{N} = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}.$$

IV. DEPENDENCIES OF λ_4

A. Dependency on the NESZ

Fig. 1 shows the dependency of λ_4 on the system noise. Calibrated data with varying levels of system noise were simulated. In this case, λ_4 is a direct measure for the noise in the data. It will also be sensitive to noise imbalance, which has not been tested here.

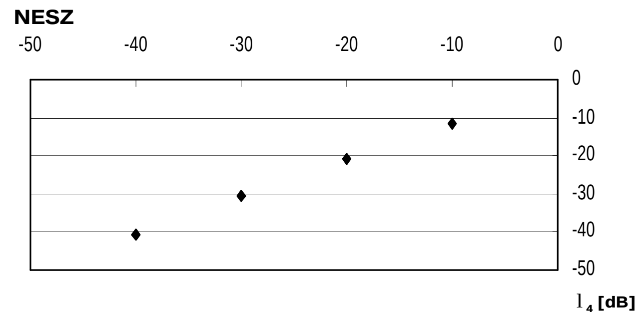

Figure 1. Fourth eigenvalue of the covariance matrix as function of the NESZ (calibrated data)

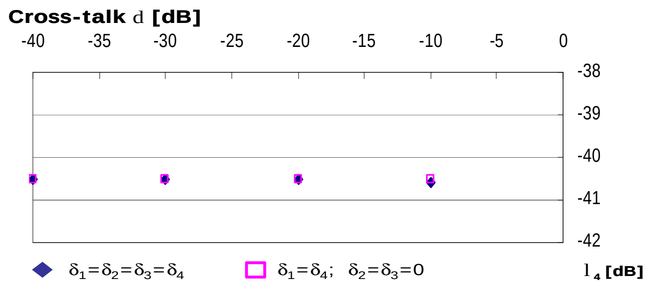

Figure 2. Fourth eigenvalue of the covariance matrix as function of cross-talk (NESZ = -40dB)

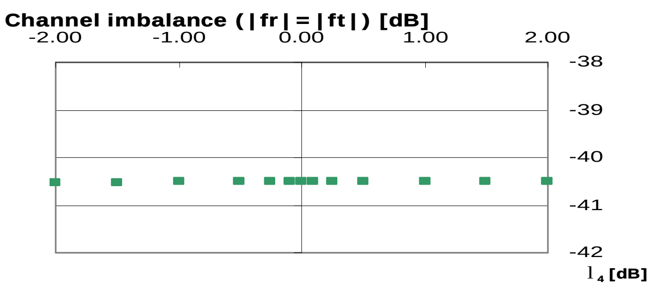

Figure 3. Fourth eigenvalue of the covariance matrix as function of channel imbalance (NESZ = -40dB)

B. Dependency on data calibration

To assess the dependency of λ_4 on data calibration, cross-talk and channel imbalance were introduced to calibrated data with a NESZ of -40 dB. Figs. 2 and 3 show that λ_4 is neither affected by cross-talk nor by channel imbalance.

This can mathematically be explained by rewriting the calibration model as follows

$$\mathbf{k}_Z \cdot \mathbf{k}_Z^+ = \mathbf{A} \cdot \mathbf{k}_S \cdot \mathbf{k}_S^+ \cdot \mathbf{A}^+$$

where

$$\mathbf{A} = \begin{pmatrix} 1 & \delta_2 & \delta_4 & \delta_2\delta_2 \\ \delta_1 & f_t & \delta_1\delta_4 & f_t\delta_4 \\ \delta_3 & \delta_2\delta_3 & f_r & f_r\delta_2 \\ \delta_1\delta_3 & f_t\delta_3 & f_t\delta_1 & f_t f_r \end{pmatrix}$$

$\mathbf{k}_z$ and $\mathbf{k}_s$ are vectorized versions of **Z** and **S** respectively. **A** combines the distortion matrices **R** and $\mathbf{T}_r$.

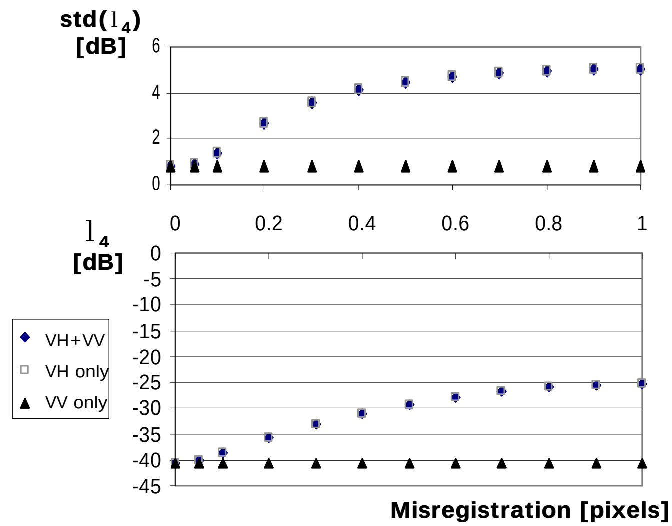

Figure 4. Fourth eigenvalue of the covariance matrix and its standard deviation as function of misregistration (calibrated data)

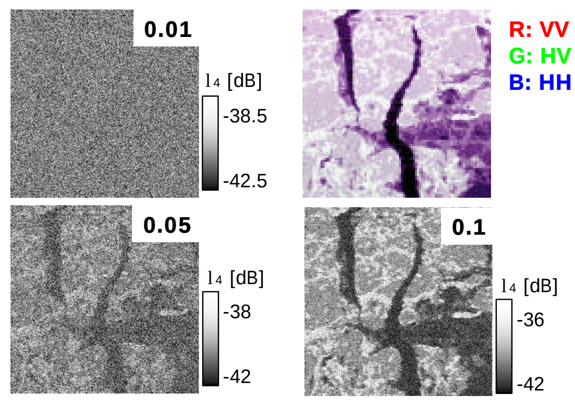

Figure 5. Image examples of the fourth eigenvalue of the covariance matrix as function of misregistration (calibrated data)

A is the identity matrix if the system is perfectly calibrated. Under regular conditions, f_r and f_t are close to one and δ_1-δ_4 are close to zero. In both cases, A is a rank 4 matrix and there will not be a change of rank for $k_Z \cdot k_Z^+$ with respect to $k_S \cdot k_S^+$. With no change of rank, λ_4 continues to represent the noise in the image. The noise estimate is therefore independent of cross-talk and channel imbalance.

C. Dependency on channel registration

Another quality factor for polarimetric SAR data is channel registration. To assess the issue we have introduced a shift of up to one pixel in azimuth between the various channels. Fig. 4 shows the increase of λ_4 depending on a misregistration between the two cross-polarization channels.

More information on calibration can be obtained by visual analysis of the λ_4 image. The increase in standard deviation can be accompanied by image content becoming visible in the λ_4 image. This is illustrated in Fig. 5. Even a small misregistration of $1/20^{th}$ of a pixel leads to image content becoming visible in the λ_4 image.

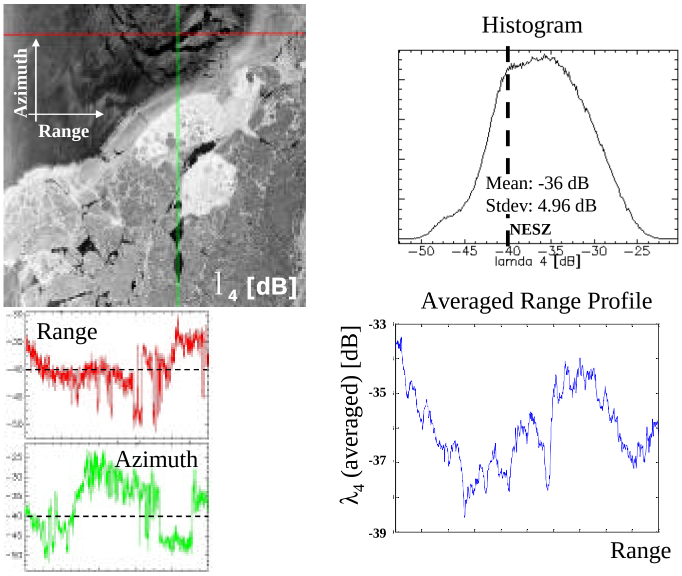

Figure 6. Fourth eigenvalue of the covariance matrix. CV-580 data of the Northumberland straight L1P3. The data were subsequently recalled due to a registration problem.

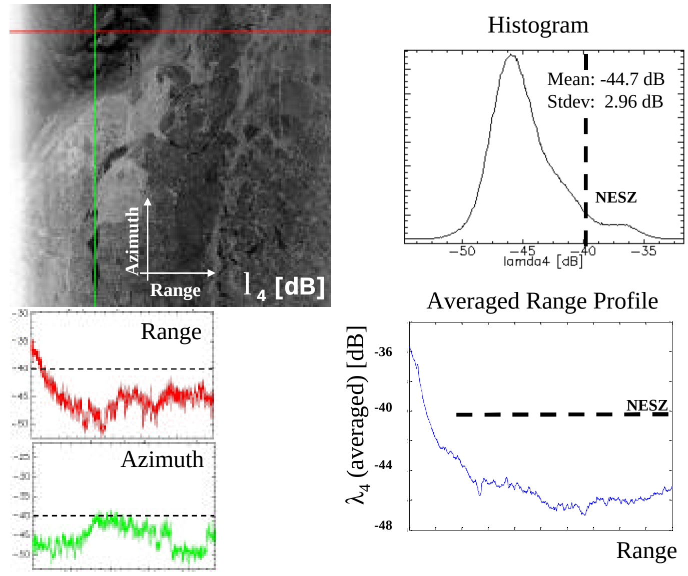

Figure 7. Fourth eigenvalue of the covariance matrix. CV-580 data of the Northumberland straight L1P3.

V. Analysis of Airborne Polarimetric SAR Data.

In March 2001, the Canadian CV-580 airborne SAR collected polarimetric C-band data in the Northumberland Straight. Several acquisition lines were made available for analysis. Of particular interest were two lines (L1P3 and L2P5) that were acquired over approximately the same region with different incidence angles. No calibration targets were present in the data.

Figs. 6 and 7 show examples of a λ_4 analysis, where a histogram, an averaged range line as well as a sample range and azimuth line are shown for each scene. Using our method, we excluded one acquisition line (L1P3) from further analysis as the levels exceeded the posted NESZ values for the system. The L1P3 data were eventually recalled by CCRS due to a channel registration problem [5].

The residual image content visible in Fig. 7 indicates a residual registration problem for the cross-polarization channels of the L2P5 data. While the data was used for polarimetric analysis, this registration issue will be subject of further investigation.

VI. Conclusions

Our experiments show that the analysis of the fourth eigenvalue of the covariance matrix can be used to identify a certain class of image quality problems. In particular, λ_4 is sensitive to:

- Misregistration of HV with respect to VH
- Image noise level (where λ_4 is a direct measure)
- Noise imbalance

No sensitivity was measured for the misregistration of one co-polarization channel up to one pixel as well as cross-talk and channel imbalance.

We have successfully used the scheme to identify data showing quality problems and exclude them from further analysis. The method is one step towards a comprehensive data quality assessment. Additional tests are required to ensure all quality criteria are met.

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